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Resampled Efficiency Issues

by

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Abstract

Once resampled efficiency gained prominence, misinterpretations and misunderstandings arose. This 2003 article addresses the most frequently asked questions and misunderstandings of that time period.

Interest in resampled efficiency has recently emerged in a number of academic and professional journals, student theses, and presentations worldwide.[1] One reason is that practitioners are now using resampled efficiency worldwide. Inevitably, misinterpretations and misunderstandings have arisen. This note addresses and resolves some of the more common misunderstandings and recurrent issues that have come to our attention.

Why resampled efficiency?

"Although Markowitz efficiency is a convenient and useful theoretical tool for defining portfolio optimality, in practice it is an error-prone procedure that often results in 'error maximized' and 'investment irrelevant' portfolios."[2] Markowitz (1991) mean-variance (MV) portfolio optimization implicitly assumes that inputs are 100% certain. But investors are never 100% certain of their information. As a result, Markowitz efficient portfolios are typically overly specific to the inputs and often lead to disappointing out-of-sample performance as well as investment unintuitive portfolios. Michaud (1998) teaches eight categories of procedures for improving portfolio optimization and asset allocation in practice including resampled efficiency.[3]

What is new about resampled efficiency?

Michaud resampled efficiency is a new definition of portfolio optimality.[4] It is a forecast certainty conditional generalization of the Markowitz MV efficient frontier. Resampling is a powerful statistical procedure for dealing with estimation error in optimization inputs. Resampled efficiency controls estimation error by allowing the user to customize the optimization process according to an assumed level of information certainty.[5] Unlike earlier work that focused on fuzzy sets of optimized portfolios, resampled efficiency defines a new efficient frontier that is consistent with most applications of MV efficiency.[6]

What are the benefits of Michaud resampled efficiency?

Michaud resampled efficiency consists of the resampled efficient frontier and the resampled rebalancing rule. The patented resampled efficient frontier (Michaud 1998, Ch. 6) is provably effective at improving investment value, leads to portfolios that are intuitive and stable, reduces the need to trade, does not require *ad hoc* constraints, and is generally much easier to manage than MV efficiency.[7] The patented resampled rebalancing rule (Michaud 1998, Ch. 7) is the first statistically rigorous portfolio-trading rule available to the investment community.[8] In general the rebalancing rule leads to a

[1] Resampled efficiency, described in Michaud (1998), was co-invented by Richard Michaud and Robert Michaud and is a U.S. patented procedure, #6,003,018, December 1999, patent pending worldwide.
[2] Michaud (1998, p. xiv).
[3] For example, Chs. 8 and 11 discuss Stein and Bayesian methods.
[4] Michaud (2003).
[5] Ten levels of input certainty are available that have been calibrated to facilitate the user experience.
[6] For example, the fuzzy region in Michaud (1989).
[7] Unlike Michaud resampled efficiency, many current proposals for resampled optimization are actually ineffective at enhancing out-of-sample investment value.
[8] See the comment in Buetow, G., et. al. (2002).

reduced need to trade. The procedure provides a reliable method for deciding when trading is or is not advisable and is customizable for a wide range of applications.[9]

Will a back test of resampled efficiency always outperform MV efficiency?
No. Rigorous statistical simulation tests show that Michaud resampled efficiency on average improves out-of-sample reward-to-risk ratios relative to MV efficiency. This proof is not a back test. A back test is period specific and provides no reliable out-of-sample investment information. Just as the best team does not always win, a "good" investment strategy may perform poorly and a "poor" strategy may perform well in a given period. This is why simulation methods are used to prove the investment superiority of resampled efficiency (Michaud 1998, Ch. 6). Intuitively, a simulation study is a controlled experiment to reliably assess how a strategy performs on average. In many cases, the proper interpretation of back tests is that they are less indicative of the relative investment merits of resampled optimization than as a metric of the predictive content of the input data. We note however that even reliable marginal performance improvements relative to peers may often have investment significance.

Are Bayes methods more powerful than Michaud resampled efficiency?
No. Bayesian methods by themselves do not solve the estimation error problem because MV optimized portfolios remain overly specific to inputs. While there are many Bayesian methods to consider, important recent evidence suggests that Michaud resampled efficiency may often outperform Bayes optimized portfolios out-of-sample.[10] In general, Bayesian and resampled efficiency procedures are both important and neither is exclusive. Roughly, Bayesian methods are a way of improving the level of investment information in your optimization inputs while resampled efficiency is a better way of using whatever information you have.[11]

Is resampled efficiency useful for long-short strategies?
Yes. Academic optimization studies often assume unbounded assets. In this investment irrelevant case resampled and MV efficiency coincide. However, in practice long–short strategies assume bounds on asset weights.[12] In investment relevant cases, resampled efficiency is not equivalent to MV efficiency. Since many long-short strategies are levered strategies, the realistic risk estimation benefits of resampled efficiency are particularly important.

Are resampled efficient portfolios overly diversified?
No. There are two separable issues. 1) Resampled efficiency is forecast certainty conditional MV efficiency. The higher the certainty level, the less diversified the resampled optimal portfolio. Resampled efficient portfolios are optimally diversified relative to the chosen level of forecast certainty. 2) It is true that resampled optimized portfolios may often have statistically insignificant weights due to random outcomes in

[9] Updates of the rebalancing rule appear in Michaud and Michaud (2002).
[10] H. Markowitz (2002) personal communication.
[11] Much of our ongoing research is devoted to development of new Bayesian procedures.
[12] See Michaud (1993). New Frontier provides long-short resampled efficiency software.

the simulation process. We deal with this by computing the theoretically optimal resampled efficient frontier and then find nearest investment relevant portfolios.[13]

Are all resampled efficient optimizers superior to MV efficiency?
No. There are a number of "resampled optimization" proposals in the investment community that have important limitations relative to Michaud resampled efficiency. In all the cases we are aware of, these alternative resampling optimization methods do not produce portfolios that uniformly improve performance out-of-sample.[14] Michaud resampled efficiency is the only technology we know that enhances investment value on average across the risk spectrum for portfolio optimization in practice.

Is MV efficiency easy to beat out-of-sample?
Simple strategies such as minimum variance and equal weighting may beat alternative Markowitz M-V optimal portfolios out-of-sample in some cases. However, such solutions have very limited usefulness for asset management in practice and are not superior to Michaud resampled efficiency.

Is the asset weight distribution anomalous?
Resampled optimization can be used to estimate the asset weight distribution (Michaud 1998, Ch. 7). This information can be very useful for deciding whether a portfolio is not statistically significantly optimal. Our original procedure often led to highly skewed distributions. It is extremely important to note that this skewness is inherent in Markowitz optimization but not obvious since we only see the point estimate of the optimal portfolio. Furthermore, an enhanced resampling procedure leads to substantial reduced skewness and improved asset weight distribution estimation.[15]

Is resampled efficiency a heuristic?
Resampled efficiency is based on one of the most powerful tools in all of modern statistics.[16] Interestingly, Markowitz optimization was also called a heuristic by many of its earliest detractors.

Is resampled efficiency sensitive to asset universe specification?
Using an optimizer is not a mathematical exercise. It requires significant investment knowledge.[17] A case in point is the process of choosing the relevant asset universe and excluding non-investable assets. Poor investments, such as baseball cards and lottery tickets, are typically excluded in professional asset allocation studies. An equal weighted portfolio from a cleverly chosen asset universe may often outperform an optimized portfolio over a poorly defined asset universe. The performance benefits of resampled relative to MV efficiency may be limited in the case of poorly defined asset universes because input uncertainty may make resampled optimization relatively more prone to

[13] We have artificial intelligence methods for solving this problem, forthcoming in our software.
[14] Michaud (1998, Ch. 6) also reports that a utility function approach often used in academic papers showed limited improved performance characteristics.
[15] The new methods are discussed further in Michaud and Michaud (2002).
[16] For a classic resampling reference see Efron and Tibshirani (1993).
[17] See Michaud (1998, Ch. 12) for a discussion of a number of common optimization design errors.

selecting poor investments. Avoiding optimization design errors is important in achieving the optimal benefits of resampled efficiency.[18]

How important are liquidity considerations in an optimization?

Liquidity is often a first order factor for defining an investment meaningful portfolio optimization. Many portfolio optimization puzzles can be traced to omission of the liquidity factor. Illiquid assets often have high estimated returns and low measured risk. This is because risk is often hard to measure without advanced statistical methods. An allocation study that includes the S&P500 index and private equity or hedge funds will often lead to investment irrelevant solutions without an appropriate adjustment for relative liquidity. Liquidity is a non-linear risk factor that depends on the level of investment and asset size or float. New Frontier Advisors now includes a liquidity prior option in our asset allocation software that we recommend using whenever asset universes have heterogeneous liquidity.

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[18] For asset managers who wish to include assets that are not necessarily investable, New Frontier Advisors, LLC has developed a procedure to exclude non-investable assets from the optimal portfolio.