Reply to "Reply to 'Comment on "Markowitz versus Michaud: Portfolio Optimization Strategies Reconsidered" ' "

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In our "Comment" Michaud, Esch, Michaud (2015), on Becker, Gurtler, and Hibbeln (2015) (BGH) we noted a number of critical limitations of their study of Markowitz (1952) versus Michaud (1998) MV optimization. In particular, in a paper primarily focused on disparaging Michaud optimization, their computation of Michaud optimization was in error, invalidating all their results and indeed purpose of publication of their original paper. In addition, as we also noted, they managed to omit reference to the important Jobson and Korkie (1981) study showing that unconstrained MV optimization was much worse than equal weighting in many practical cases, invalidating two-thirds of their results that addressed unconstrained MV optimization. We also noted a number of additional limitations including that they managed to ignore published rebuttals to what they claimed were limitations of Michaud optimization.[1] We concluded they had no contemporary understanding of MV optimization in practice and could not consider any of their conclusions credible.

BGH (2017) published an update to their paper, which purports to address concerns raised in our response to their article, including the use of the correct Michaud optimization procedure. However, as we note below, the new response raises even more questions about the reliability of their methods, contains surprising results which contradict much well-known peer-reviewed published research, still retains evidence of flawed Michaud calculations, and remains inconsistent with issues they claim to address.

Table 1 below presents the authors' revised results of their original Table 4 with claimed corrections of their original flawed Michaud optimization methodology. Careful obsevation will note not only the Michaud results are changed, so are other results that have nothing to do with Michaud optimization. The table summarizes the values of their Φ_1 score, a simple tally of wins for their "preference function," which we note to be identical to the standard quadratic utility of mean minus risk aversion parameter times variance.[2] We infer that their notion of "statistical significance," although unspecified in their paper or reply, corresponds to a statistical test bounding the score away from 50%. This must be true because the only values presented without significance at 1% are close to 50%. Normally, highly significant scores represent a high degree of certainty in their assertion of superiority of one method over another, yet in a few cases the winner is different in the revision than it was in the original, for methods that were not revised and should produce consistent results in a well-designed reliable simulation study. Of the 21 Φ_1 scores presented in the upper left quadrant of the table, two of them (C vs FS and J vs FS) have reversed direction! These revised results, moreover, contradict some of the qualitative findings of their original

[1] The most serious published critique of Michaud optimization, Harvey et al (2008), is also flawed as noted in Michaud and Michaud (2008c) and as Harvey et al (2008b) acknowledge.

[2] BGH claimed that we mistakenly thought they were using a quadratic utility function. We see no difference.

paper! In the abstract of the original article, the authors recommend the Frost-Savarino estimation method because it wins against all competitors, yet their revised findings reverse two of the six wins without explanation or comment, greatly weakening this finding. This change to some their most significant outcomes calls into serious question the reliability of their simulation methods for all their results.

Indeed, without discussion, many of their revised findings present contrary results to many of their cited sources, including Garlappi, Uppal and Wang (2007), Jobson and Korkie (1981), Jorion (1986), and Frost and Savarino (1986). Their Table 1 shows the estimation method denoted “C”, which consists of using the historical sample means, variances, and correlations, beating equal weight, minimum variance, both Ledoit-Wolf methods, and Frost-Savarino (contrary to the results described in their original work). These findings are contrary to much other published work which show out-of-sample benefits to these enhanced estimation methods compared to simple historical estimation. All of the aforementioned authors offer various testimonies to the inferiority of the sample mean and variance for purposes of Markowitz optimization. Of course it it possible to design an experiment, by choice of simulation parameters and risk aversion parameter, that will favor any method, but the limits of the experiment should be discussed more thoroughly so that the reader will draw appropriate conclusions.

Their new results still raise serious doubts that they are now correctly calculating Michaud optimization. Consider the Michaud case for “EW” estimation. Simulations should show it is identically an equal-weighted portfolio, as is also true for Markowitz portfolio. There is no way for either portfolio to “win” over the other since they are identical. Their result table for both original and revised Michaud methods shows a statistically significant departure from 50% for the win rate, which would suggest that the portfolios are, once again, not being calculated correctly, or that their entire simulation study framework is far too unstable to reach reliable conclusion in any instance. In fact this particular notion of “statistical significance” in a simulation study is somewhat arbitary when increasing the number of simulations can increase the significance of any table entry that is not identically 50%. In this setting everything is as significant as the experimenters want it to be, provided enough simulations are run.

Michaud (1998, Ch. 6) and Michaud and Michaud (2008a Chs. 6,9, b) provide carefully detailed simulation studies to show that Michaud optimization provides superior out-of-sample investment value under situations of practical investment interest. No less an authority than Markowitz has concluded similarly. The often misuderstood Markowitz and Usmen (2003) study was designed not to show that Markowitz optimization was superior to Michaud but to show that superior information and Markowitz should be superior to Michaud without superior information. As the study showed, Michaud was superior to Markowitz even with inferior information.

As a final comment, the entire structure of the BGH report is based on a simple but fatal misuderstanding. Michaud (1998, Chs. 8, 9, 12) and Michaud and Michaud (2008, Chs. 8, 9, 12) discuss at length the importance of using various estimators of risk and return, such as Ledoit, Frost and Savarino, Stein, and other methods to add value to portfolio optimization in practice. Optimization and estimation are independent procedures. The interesting question is not whether Frost and Savarino beats Michaud, it is whether Frost and Savarino improves Michaud over pure history.

To summarize, their results in an extremely well understood example of Michaud and Markowitz make no sense, their study appears to have serious instability and unreliablity given that new results are often inconsistent with original ones for cases where there should be no change, and that their study is based on a misunderstanding of optimzation versus estimation. We do not recommend trust in the general applicability of any of the conclusions of their study.

Table 1. Comparison of different approaches in the "no borrowing and lending" case

		Markowitz							Michaud						
	↓ wins against →	C	EW	MV	LW (GV)	LW (SI)	J	FS	C	EW	MV	LW (GV)	LW (SI)	J	FS
Markowitz	C	-	99,5 ***	98,0 ***	99,0 ***	97,9 ***	39,7 ***	60,0 ***	68,8 ***	99,5 ***	99,3 ***	99,5 ***	98,6 ***	74,1 ***	72,3 ***
	EW	0,5 ***	-	51,3 *	47,2 ***	51,6 **	0,4 ***	0,4 ***	0,2 ***	47,8 ***	59,0 ***	51,0 ***	56,0 ***	0,3 ***	0,1 ***
	MV	2,0 ***	48,7 *	-	45,8 ***	49,9	1,7 ***	1,8 ***	1,5 ***	48,7 *	53,0 ***	49,2	51,4 ***	1,8 ***	2,3 ***
	LW (GV)	1,0 ***	52,8 ***	54,2 ***	-	53,5 ***	0,8 ***	1,0 ***	0,6 ***	52,8 ***	58,7 ***	55,0 ***	54,9 ***	0,6 ***	0,7 ***
	LW (SI)	2,1 ***	48,4 **	50,1	46,5 ***	-	1,9 ***	1,9 ***	1,7 ***	48,4 **	53,0 ***	49,2	51,8 ***	2,0 ***	2,2 ***
	J	60,3 ***	99,6 ***	98,3 ***	99,2 ***	98,1 ***	-	58,0 ***	69,4 ***	99,6 ***	99,4 ***	99,6 ***	98,8 ***	75,2 ***	71,7 ***
	FS	40,0 ***	99,6 ***	98,2 ***	99,0 ***	98,1 ***	42,0 ***	-	59,5 ***	99,6 ***	99,5 ***	99,6 ***	98,8 ***	65,3 ***	79,9 ***
Michaud	C	31,3 ***	99,8 ***	98,5 ***	99,4 ***	98,3 ***	30,7 ***	40,6 ***	-	99,8 ***	99,6 ***	99,8 ***	99,0 ***	75,6 ***	63,6 ***
	EW	0,5 ***	52,2 ***	51,3 *	47,2 ***	51,6 **	0,4 ***	0,4 ***	0,2 ***	-	59,0 ***	51,0 ***	56,0 ***	0,3 ***	0,1 ***
	MV	0,7 ***	41,0 ***	47,0 ***	41,3 ***	47,0 ***	0,6 ***	0,5 ***	0,5 ***	41,0 ***	-	37,0 ***	50,0	0,7 ***	0,4 ***
	LW (GV)	0,5 ***	49,0 ***	50,8	45,0 ***	50,8	0,4 ***	0,4 ***	0,2 ***	49,0 ***	63,0 ***	-	57,0 ***	0,3 ***	0,0 ***
	LW (SI)	1,4 ***	44,0 ***	48,6 ***	45,1 ***	48,2 ***	1,2 ***	1,2 ***	1,0 ***	44,0 ***	50,0	43,0 ***	-	1,1 ***	1,2 ***
	J	25,9 ***	99,7 ***	98,3 ***	99,4 ***	98,0 ***	24,8 ***	34,7 ***	24,5 ***	99,7 ***	99,4 ***	99,7 ***	98,9 ***	-	56,5 ***
	FS	27,7 ***	100,0 ***	97,7 ***	99,3 ***	97,8 ***	28,3 ***	20,1 ***	36,4 ***	100,0 ***	99,6 ***	100,0 ***	98,8 ***	43,5 ***	-

Note: This table presents the performance index Φ_1 (in %) for the "no borrowing and lending" case and an observation period of 60 months.
* Statistical significance at the 10% level.
** Statistical significance at the 5% level.
*** Statistical significance at the 1% level.

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