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Optimal and Investable Portfolios

by

Richard O. Michaud and Robert O. Michaud

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Abstract

Optimal portfolios typically include inconvenient and insignificant asset weights, make for impractical investment. This article introduces some of NFA's compute-efficient solutions for finding an investable portfolio from the optimal portfolio.

Optimized portfolios and asset allocations are not investable. Optimized portfolios typically have asset weights that are not in convenient units for investment or are investment insignificant. Current methods for converting optimal into investable portfolios are generally ad hoc and may not lead to desirable investment properties.

In this note we examine the issue of portfolio optimality versus investability. Our results shed new light on the notion of diversification in an optimal portfolio. In particular, a theoretically optimal portfolio will typically include investments in all or nearly all attractive investments. Many of the optimized asset weights, however, will have insignificant economic value. To solve this problem we introduce an important, new, compute-efficient, and practical procedure for converting an optimized portfolio into an investable portfolio.[1]

Rounding and Optimized Portfolios

Institutional asset managers typically make investments in large dollar increments or in round lots of stocks. Standard portfolio optimizers, on the other hand, typically produce fractional asset weights. Fractional weights exist in part because portfolio optimizers are general-purpose numerical analysis solvers not designed specifically for investment management.[2]

Investors convert optimized asset weights into convenient investment units to define an "investable" portfolio. Large fractional weights are rounded, and weights too small for investment, which we call nuisance weights, are ignored. The rounding process defines investability, not optimality. As long as the process is not too extensive, the optimality of the investable portfolio may not be adversely affected.
In the following, we further examine the investment meaning of nuisance weights and their implications for optimal diversifications and present New Frontier's new procedure for converting optimal into investable portfolios.

Portfolio Construction Basics

An active investment process begins by identifying attractive assets from a given asset universe. The next step is to form a portfolio of the attractive assets. Naïve portfolio formation solutions, common among some traditional active managers, are to equal or benchmark weight the attractive assets.[3] Note that these rules result in investments in all the attractive assets in a very sensible way.

From the point of view of modern investment theory, naïve portfolio formation rules have severe limitations. They do not control for desired level of portfolio risk or reward-

[1] The algorithm was developed by Robert Michaud and other New Frontier Advisors, LLC (NFA) associates. The option is available in the recently released version 3.5 of NFA's Asset Allocation Suite.
[2] Integer programming or other discrete optimization methods are the exception where the investing unit constraint is a part of the definition of portfolio optimality. Among other limitations, discrete methods tend to be very slow and impractical for many investment applications.
[3] Some obvious variations on the theme include equal weighting within sectors or industries and sector or industry weighting a stock portfolio. Note that benchmark weighting refers to weighting the attractive assets where the weights are redefined to satisfy the budget constraint.

to-risk ratio optimality.[4] On the other hand, standard optimized portfolios often omit investment in many of the attractive assets, a characteristic that traditional managers may consider undesirable. Is investment theory inconsistent with investing in all attractive investments?

Portfolio Optimization

The standard in institutional practice for defining portfolio optimality is Markowitz, or classical, MC efficiency.[5] As discussed in previous newsletters, Markowitz efficiency is far more robust and useful than is widely understood, as well as convenient and easy to use for many investment purposes.[6]

Markowitz efficiency, however, as poor out-of-sample performance characteristics and often leads to unintuitive and unmarketable portfolios.[7] The limitations of Markowitz efficiency are due to the fact that input information is assumed to be 100% certain, a condition never met in practice. Investors always have some uncertainty associated with their inputs. Michaud resampled efficiency controls estimation error by allowing the user to choose an appropriate certainty level to condition the optimization process.[8,9] Performance on average is improved out-of-sample.

Resampled Optimization and Diversification

A characteristic of resampled optimization unlike classical optimization is that all attractive assets are often included in a resampled optimal portfolio. Resampled optimized portfolios are averages of simulated MC optimal portfolios computed from many simulated MC efficient frontiers. While practical considerations limit the number of computed simulated efficient frontiers for defining the Resampled Efficient Frontier™, there is theoretically no limit to how many simulations may be performed. In the limit, any asset deemed attractive is likely to be in one of thousands of possible simulated efficient frontiers. In theory, virtually all attractive assets have a weighting in some Resampled Efficient Frontier™ portfolio.

Note that investment in all attractive assets is consistent with naïve portfolio formation rules. Resampled efficient portfolios, however, are also risk controlled and reward-to-risk ratio optimal, albeit at the cost of more nuisance asset weights.

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[4] The exception is the zero residual risk case where the portfolio includes all the assets benchmark weighted.
[5] Markowitz (1959).
[6] Levy and Markowitz (1979).
[7] See Michaud (1989).
[8] Resampled efficiency, described in Michaud (1998), was co-invented by Richard and Robert Michaud. It is a U.S. patented procedure, #6003018, December 1999, patent pending worldwide. See also Michaud (2003). Various updates are available at www.newfrontieradvisors.com. The term "Michaud resampled efficiency" is occasionally used to note that NFA's optimization technology has very significant investment dissimilarities with other "resampled" optimization methods. New Frontier Advisors, LLC has exclusive worldwide licensing rights.
[9] Ten levels of input certainty are available that have been calibrated to facilitate the user experience.

Investable versus Optimal Portfolios

Optimal portfolios generally need to be converted into investable portfolios. The rounding heuristic is one approach. However, this is a by-hand procedure that is unlikely to be investment optimal since it does not consider correlations or tradeoffs with risk and return.

Approaches using mixed integer or other discrete programming methods also have serious limitations. Because investability constraints are part of the optimization process, the portfolio is not investment optimal and may have undesirable investment characteristics. In particular, the post optimization performance benefits of resampled efficient portfolios may not be present with such alternative procedures. What is desirable is to compute investable portfolios with the investment benefits of resampled efficient portfolios using a convenient and efficient procedure.

NFA Post Optimization

NFA's post optimizer is a new compute-efficient and convenient algorithm to optimally convert resampled efficient into investable portfolios.[10] The three categories of investability options are cardinality, threshold, and increments or round lots.

Cardinality refers to the number of desired assets in the post optimized portfolio; e.g. a cardinality of five finds optimal allocations for the five most important assets in the resampled optimal portfolio. Threshold refers to the minimum non-zero asset weight; e.g. a threshold of 4% implies a minimum weight of 4% in any post optimized asset. Increments, which can be thought of as round lots, refer to the minimum additional asset weight allowed; e.g. an increment of 1% means that all post optimized asset weights are in units of 1%.

The following illustrates the NFA post optimizer for a specific example. The optimization inputs are taken from the six countries and two bond indices example in Michaud (1998). The post optimization options are: cardinality = 6, threshold = 4%, increment =1%. A middle level of forecast certainty and 500 simulations are used to compute the resampled optimal portfolio. The first column in the table provides the names of the country and bond indices. The second column provides the resampled optimal portfolio asset allocations roughly midway up the Resampled Efficient Frontier™. The third column provides the associated post optimized allocations.

Here we see that Canada and US Bonds have been eliminated as the two least significant assets, and the remaining six assets have increased in weight. This is sensible because the remaining assets are all positively correlated to the eliminated assets. Most of the increases are small, with the notable exception of Euro Bonds, which increases nearly as much as the previous allocation to US Bonds. Note that US Bonds had a much larger weight than Germany. However, Euro Bonds are an excelled substitute for US Bonds at this risk level, while German equities are not so easily approximated.

[10] The procedure is available on the Analysis Worksheet of the Optimizer module in version 3.5 of NFA's Asset Allocation Suite.

Asset Names	Resampled Optimized	Post Optimized
Canada	2.0%	0.0%
France	10.2%	11.0%
Germany	5.4%	6.0%
Japan	15.5%	16.0%
UK	10.2%	11.0%
US	26.3%	27%
US Bonds	8.1%	0.0%
Euro Bonds	22.2%	29.0%

The post-optimize option computes investable portfolios according to the investability options for all the portfolios on the Resampled Efficient Frontier™. The new algorithm uses artificial intelligence methods and heuristics tuned to the resampled optimal portfolio investability problem. The end result is a highly compute-efficient, yet practical and convenient, algorithm for many problems of investment interest. We believe NFA's procedure is the most theoretically rigorous and useful available to the investment community today.

References

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