NEW FRONTIER ADVISORS, LLC

Forecast Confidence Level and Portfolio Optimization

by

Richard O. Michaud and Robert O. Michaud

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Abstract
This report focuses on the role and importance of the uncertainty in forecast information in constructing portfolios with the optimal performance.

The purpose of a portfolio optimizer is to enhance investment performance by optimally using information. Unfortunately, optimizers are well known to create portfolios that perform poorly even when compared to equal weighted portfolios. Perhaps surprisingly, bad luck and poor estimates are not generally the primary reason for poor performance. The essential problem is that portfolio optimizers do not properly treat the uncertainty implicit in all investment information. Optimizers assume that optimization inputs are 100% certain. But investors are never 100% certain of future estimates of risk and return. This is why optimized portfolios don't make investment sense to experienced investors and don't have investment value.

The focus of this report is to discuss the role and importance of the uncertainty in forecast information in constructing more optimally performing portfolios. Monte Carlo or resampling methods enter naturally as a means of dealing with information uncertainty. Resampled Efficiency™ is a generalization of Markowitz (1959) mean-variance (MV) portfolio optimization that properly includes investment uncertainty in the optimization process.[1] The end result is a very substantial increase in investability, performance, and ease of use.[2]

Coin Tossing and Uncertain Forecasts

Consider forecasting the number of heads in ten tosses of a coin. What uncertainty exists in this situation?

Case 1: The coin is known to be fair.
Since five heads is the average and most likely number, five heads is the obvious forecast. However, there is significant uncertainty that your forecast is correct. This is because any number of heads from zero to ten may occur when the coin is actually tossed. In fact there is a high probability that you will be wrong in any given play of the game. Only if you are able to play the game many times is your forecast, on average, likely to be safe.

Case 2: It is uncertain that the coin is fair.
A reasonable "no-reliable-information" forecast is five heads. Unless the coin turns out to be very biased the five-heads forecast is as good as any other. But you are less certain of your forecast than in Case 1. Not only can any number of heads from zero to ten actually occur in any play of the game, the coin may not be fair and, on average, no forecast is very safe.

Investment Uncertainty

In investment practice it is Case 2 that represents investor uncertainty. Both the actual outcome conditional on expectations, and uncertainty of the reliability of expectations, exists in asset management forecasts of future returns.

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[1] Resampled Efficiency is described in Michaud (1998, Ch. 6). It was co-invented by Richard Michaud and Robert Michaud, U.S. patent December (1999). Worldwide patents pending. New Frontier Advisors, LLC (NFA) is the exclusive worldwide licensee.
[2] Introductory materials, advanced discussions, and new research results and developments can be found at www.newfrontieradvisors.com.

Asset management uncertainty is compounded by the amount that exists even in the unrealistic case when future risks and returns are assumed known. To illustrate, assume a correct forecast of 10% return and 20% standard deviation for the stock market. The range of possible returns is roughly -10% to 30% two-thirds of the time. This is a very wide range of outcomes that does not take into account the uncertainty of the forecast or likelihood of outlier events. In general, investors face a great deal of uncertainty when making investment decisions.

Portfolio Uncertainty

Long-term U.S. capital market returns show that, on average, stocks outperform bonds and have more volatility. Assume for simplicity that historical returns imply a 10% return and 20% standard deviation for stocks, a 5% return and 10% standard deviation for bonds, and a 0.5 correlation.

Case 1: It is known that long-term historical data reliably represent future returns. The investor forecasts stocks outperforming bonds. This is because stocks have roughly a 60% probability of outperforming given our assumptions. But there is much uncertainty in this forecast. There is a large probability that bonds will outperform stocks in any given investment period making a portfolio devoted 100% to stocks seem very imprudent. As a consequence investors typically diversify investable assets into both stocks and bonds. A 60/40-asset mix of stocks and bonds is a very common recommendation by institutional consultants and professional financial planners.

Case 2: It is uncertain that long-term historical data reliably represent future returns. In this case a more diversified stock/bond portfolio than that in Case 1 seems advisable. At the extreme of uncertainty an investor may wish to be highly diversified by equal weighting stocks and bonds.

The effect of Case 1 uncertainty is to rationalize portfolio diversification in both stocks and bonds. The effect of Case 2 uncertainty is to rationalize increased diversification relative to Case 1. As will be shown below, Resampled Efficiency includes both Case 1 and 2 levels of uncertainty in the optimization process. The Resampled Efficient Frontier™ produces portfolios that are far better diversified than MV optimized portfolios for any given set of inputs. In addition, Resampled Efficiency allows the investor to control uncertainty relative to the perceived information reliability level. As in the example for stocks and bonds, Resampled Efficiency portfolio diversification increases as investor uncertainty in the reliability of the inputs increases.

Markowitz MV Efficiency

Markowitz efficiency has been the standard for portfolio optimization technology in investment practice for nearly fifty years. Markowitz optimization has many important theoretical and practical properties. MV optimization is a useful approximation for maximizing expected utility under a wide variety of practical investment situations. MV optimization is also a useful approximation for multi-period and continuous-time optimization and asset-liability optimization. Very generally, conventional alternatives to MV optimization have no fewer limitations.

However, MV optimization is well known to have severe performance limitations in practice. Equal weighted portfolios may often outperform MV optimized portfolios. The problem is that MV optimization treats inputs as point estimates. Case 1 and 2 investor uncertainties are ignored. Any other risk and return scenario in the investment period than the one forecast is likely to lead to poor performance.

An ideal solution is an optimization process that inherits the theoretical benefits of Markowitz MV efficiency while addressing its limitations relative to information uncertainty. Happily, Resampled Efficiency is a generalization of MV efficiency that addresses both Case 1 and 2 uncertainties.

Case 1: Uncertainty and Resampled Efficiency

Resampled Efficiency optimized portfolios don't depend on observing the forecast scenario in the investment period. Resampled Efficiency portfolios are defined to be optimal relative to the many ways assets and markets may perform based on the optimization inputs.

The procedure starts by Monte Carlo simulating many returns consistent with the optimization inputs. Statistically equivalent estimates of risks and returns are computed from a given set of simulated returns. Many sets of simulated returns are computed to form many statistically equivalent estimates of risk and return. A simulated MV efficient frontier is computed for each set of simulated risks and returns. The Resampled Efficient Frontier portfolios are an average of properly associated portfolios on each of the simulated MV efficient frontiers. The Resampled Efficient Frontier is a plot of the portfolio averages in terms of the original risk and return estimates.

Resampled Efficiency optimized portfolios include the uncertainty implicit in estimates of risk and return. Each simulated MV efficient frontier is an example of optimal portfolios relative to a way that assets and markets may perform conditional on the original forecasts. The efficient frontier resampling process is exactly analogous to tossing the coin ten times many times in Case 1 and observing the results. As a consequence, Resampled Efficiency optimized portfolios are not specific to any one scenario of risk and return estimates but are an average relative to many possible scenarios. That is why Resampled Efficient Frontier portfolios perform better, on average, than MV optimized portfolios. Also, because Resampled Efficiency optimized portfolios more naturally reflect investor uncertainty, they tend to be more investment intuitive to experienced investors.

Case 2: Uncertainty and Resampled Efficiency

Does Resampled Efficiency address only Case 1 estimate uncertainty? An average of properly associated MV efficient frontier portfolios does not consider investor uncertainty relative to the level of reliability of risk and return estimates. We introduce the concept of Forecast Confidence (FC) level in order to address Case 2 uncertainty in Resampled Efficiency.[3]

Suppose that risk and return estimates are based on a hundred years of monthly return data. An investor may feel very certain that these estimates reliably describe the true risks and returns for these assets over this time period. On the other hand, risk and return estimates based on a small number of monthly returns are less certain to represent the true risks and returns. So the number of returns representing the data is a measure of level of certainty.

While the concept of associating the number of returns in a data set with a level of certainty is intuitively appropriate, it is not a very useful measure when forecasting the future. The last one hundred years are not likely to be a very accurate representation of the risks and returns observed for next year, the next five years, or even the next hundred years.

The key to imbedding Case 2 information uncertainty in the resampling optimization process is to note that the number of returns computed to form a set of simulated risks and returns for each simulated MV efficient frontier is a free parameter. The number of simulated returns can be made to vary according to a view of the level of reliability in the information. A large number of simulated returns is consistent with more certainty; a small number of simulated returns is consistent with less certainty. An FC level is defined by the number of simulated returns used to compute each set of simulated estimates of risk and return.

We have calibrated ten FC levels to facilitate the user experience. Level one represents very low certainty; level ten represents very high certainty. In this framework Markowitz optimization is an eleven (absolute certainty) and total uncertainty (equal or benchmark weighting) a zero. As the FC level is increased, Resampled Efficient Frontier portfolios are less diversified and use information more actively. At the extreme, Resampled Efficiency is Markowitz efficiency. At the other extreme of complete uncertainty, Resampled Efficiency is the same as equal or benchmark weighting.

Importance of Uncertainty in Portfolio Optimization

Resampled Efficiency addresses both components of investment uncertainty inherent in asset management. In contrast Markowitz MV efficiency assumes perfect certainty in risk and return estimates. As a consequence, it should not be surprising that Resampled Efficiency is a dominant procedure relative to Markowitz efficiency.

Simulation tests performed in Michaud (1998, Ch. 6) showed that Resampled Efficiency improves return or reduces risk on average relative to MV optimization. Markowitz and Usmen (2003) showed that Resampled Efficiency improves return or reduces risk relative to MV optimization even with substantially inferior risk and return estimates. Properly

[3] Patent pending.

considering uncertainty is the key that results in Resampled Efficiency's many desirable investability and productivity properties relative to MV optimization.

Applications of FC Levels

The FC option makes the optimization process a more congenial experience for many asset managers because it addresses important investment considerations omitted in traditional optimization. As importantly, FC provides new investment tools and capabilities.

Dynamic Optimized Portfolio Management

The FC can be thought of as an accelerator pedal for the portfolio management process. Markets and economic factors don't wait conveniently for the end of the month or quarter to change direction. As the market outlook changes, investment uncertainty may wax or wane. The FC option can be used to dynamically manage optimized portfolios according to changes in the market outlook without requiring changes in estimates.

Single-Period and Multi-Period Forecasts

In many asset allocation applications, MV optimization inputs often reflect multi-period forecasts of risk and return. On the other hand, MV optimization is theoretically a single-period optimization framework. Reconciling multi-period applications of the single-period optimization framework has been an outstanding conundrum for asset managers for many years. This conundrum is resolvable using FC.

Assume that U.S. capital market return history is relevant for forecasting future risks and returns. A forecast of stocks outperforming bonds has a high level of certainty for a ten-year investment horizon. In this case a relatively high FC level may be appropriate for finding optimal asset allocations. On the other hand, the uncertainty of stocks outperforming bonds for a one-year investment horizon is greater and a lower FC level may be more appropriate. The FC option helps to bridge the gap between multi- and single-period applications of Resampled Efficiency optimization.

As a general rule, investors may find that increasing the FC level as the length of the forecast investment horizon increases is appropriate all other things the same. However, FC levels are also related to views of investment information reliability for any given investment horizon. It is worth noting that the investment implications of wrong long-term investment horizon forecasts may be more serious and may affect the appropriateness of a high FC level in some cases. Note also that for asset classes with less reliable capital market history or investment theory, the FC option may have limited association with investment horizon length.

An Alternative TAA Strategy

In a traditional tactical asset allocation (TAA) strategy, an asset manager varies risk by moving up or down an efficient frontier relative to the market outlook. For example, some TAA strategies range episodically from a high percent in stocks to a high percent in fixed income securities. Many institutional consultants are less than enthusiastic about recommending TAA strategies. They are concerned that the episodic swings of the strategy represent poor risk control and may often result in poor investment performance. On the other hand, asset managers often counter that not allowing some flexibility in using market outlook information leads to unnecessarily suboptimal investment performance.

The FC option allows for an entirely new implementation of TAA that may reconcile these competing points of view. Instead of changing the portfolio risk of an efficient portfolio, vary the FC level while holding portfolio risk constant. In this case the strategy represents moving vertically either up or down across Resampled Efficient Frontiers with different FC levels instead of riding up and down a single efficient frontier.

An FC based TAA implementation has attractive investment characteristics. TAA investment information is not ignored and has the potential of improving performance while maintaining a constant level of efficient portfolio risk. While the range of returns available from an FC implemented TAA strategy is less, the tradeoff in reliability and confidence in results may often be attractive for sophisticated investors.

Summary

Investment uncertainty is inherent in all asset management in practice. Properly addressing investment uncertainty is the key to effective and practical portfolio optimization. MV optimization has been the standard for portfolio optimization in practice for the last fifty years. While theoretically attractive and computationally convenient MV optimization is well known to have poor investment performance properties out-of-sample. The limitations of MV optimization as a practical investment tool are the consequence of ignoring the components of uncertainty in investment information: 1) the many ways markets and assets may perform conditional on expectations, 2) the level of reliability of expectations. Resampled Efficiency is a generalization of MV efficiency that addresses both components of information uncertainty in portfolio optimization. Resampled Efficiency includes a new investment tool – Forecast Confidence level – that provides a more congenial and useful asset management framework, leads to new investment strategies, and resolves outstanding conundrums. Resampled Efficiency results in investment intuitive portfolios, stable and unambiguous investment decision-making, and substantially enhanced investment performance.

References

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