Estimation Error and the "Fundamental Law of Active Management"
Is Quant Fundamentally Flawed?

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According to widely referenced applications of Grinold (1989) "Fundamental Law" theory, simply adding securities to an optimization universe, adding factors to a forecast return model, trading more frequently, or reducing constraints can add investment value to an optimized investment strategy. We show with intuitive discussion followed by Monte Carlo simulation that many applications of Grinold theory for optimized portfolio design are often unreliable and self-defeating. Critical limitations of the theory are due to ignoring estimation error (Michaud 1989) and constraints required in practical applications. A substantial fraction of professional actively managed funds may be negatively impacted.

Keywords: active management, portfolio construction, asset allocation, quantitative finance, Fundamental Law of Active Management, optimization

Highlights:

1. Estimation Error cannot be ignored as a dangerous source of underperformance for quantitative managers, especially when mean-variance optimizers are used to construct portfolios. A substantial proportion of actively managed funds may be impacted by neglecting estimation error.
2. The four implied principles of management often taught as corollary to Grinold's "Fundamental Law of Active Management, (1) More assets in the investment universe, (2) More factors used in forecasting, (3) More frequent trading, and (4) Removing constraints from optimizations, are not necessarily additive in the presence of estimation error, and can harm out-of-sample performance when applied aggressively, contrary to the guidance of the Fundamental Law formulas.
3. Grinold's formulation of the Fundamental Law is often taught as fact by many institutional education programs. The real-world failure of many of the necessary conditions for the mathematical proof are seldom included in finance curricula, but should be noted.

Benchmarks arise naturally in judging asset manager competence and for meeting investment goals. An active investment manager typically claims to provide enhanced return on average relative to a given benchmark or index for a given level of residual risk. The information ratio (IR) – estimated return relative to benchmark per unit of residual risk or tracking error – is a convenient and ubiquitous framework for measuring the value of active investment strategies.

The Grinold (1989) "Fundamental Law of Active Management" asserts that the maximum attainable IR is approximately the product of the Information Coefficient (IC) times the square root of the breadth (BR) of the strategy.[*] The IC represents the manager's estimated correlation of forecast with ex post residual return while the BR represents the number of independent bets or factors associated with the strategy.

Grinold and Kahn (1995, 19990 (GK) assert that the "law" provides a simple framework for enhancing active investment strategies. While a manager may have a relatively small amount of information or IC for a given strategy, performance can be enhanced by increasing BR or the number of independent bets in the strategy. In particular, they state "The message is clear: It takes only a modest amount of skill to win as long as that skill is deployed frequently and across a large number of stocks."[†] Their recommendations include increasing trading frequency, size of the optimization universe, and number of factors of models for forecasting return. Assumptions include independent sources of information and IC constant for each added bet or increase in BR.

Clarke, de Silva and Thorley (2002, 2006) (CST) generalize the Grinold formula by introducing the "transfer coefficient" (TC). TC is a scaling factor that measures how information in individual securities is "transferred" into optimized portfolios. TC represents a measure of the reduction in investment value from optimization constraints. This widely influential article has often been used to promote many variations of hedge fund, long short, and alternative budget only constrained (unconstrained) investment strategies.[‡]

A significant literature exists on applying Grinold theory and variations for rationalizing various active equity management strategies. Extensions include Buckle (2004), Qian and Hua (2004), Zhou (2008), Gorman et al (2010), Ding (2010), Huiz and Derwall (2011), Ding and Martin (2017). Industry tutorials and perspectives include Kahn (1997), Kroll et al (2005), Utermann (2013), Darnell and Ferguson (2014), Menchero (2017). Teachings include the Chartered Financial Analyst (CFA) Institute Level 2, the Chartered Alternative Investment Analyst (CAIA) Level 1 and many conferences and academic courses in finance. Texts discussing the formula and applications include Focardi and Fabozzi (2004), Jacobs and Levy (2008), Diderich (2009), Anson et al (2012), Schulmerich et al (2015). A very substantial fraction of globally professionally

[*] The Grinold formula is analytically derived and based on a budget only constrained maximization of quadratic utility. It should not be confused with Markowitz (1952, 1959) which assumes linear (inequality and equality) constrained portfolios and requires quadratic programming techniques to compute the MV efficient frontier. In particular, the Markowitz efficient frontier is generally a concave curve in a total or residual return framework while in Grinold (see e.g., GK 1995, p. 94) it is a straight line emanating from a zero residual risk and return benchmark portfolio. The Grinold derivation also assumes IC small, in the order of 0.1.

[†] GK (1995, Ch. 6, p. 130), also GK (1999, Ch. 6, p. 162).

[‡] One example is Kroll et al (2005). Michaud (1993) was the first to note possible limitations of the long-short active equity optimization framework.

managed funds is estimated to employ optimized portfolio design principles that are applications of Grinold theory.

We show with qualitative discussion and Monte Carlo simulation that the GK and CST proposals based on Grinold theory for optimized portfolio design may often be unreliable and self-defeating. Applications are based on a theory that assumes no estimation error in plays of the investment game as in roulette in a casino.* Unlike roulette, the investment game signal is not constant and may often be negative. As we show in Exhibit 1 in our simulation study, there is an enormous difference how optimized strategies perform on average out-of-sample when estimation error is assumed (Michaud 1989). Our simulation results generalize the classic Jobson and Korkie (1981) (JK) and Frost and Savarino (1988) (FS) estimation error simulation studies and rationalize the empirical "1/N" results as in deMiguel et al (2007).† Applications of the GK and CST precepts of active management and associated published papers and texts have gone largely unchallenged for more than twenty-five years. The limitations we identify have likely negatively affected the performance of a substantial fraction of globally professionally managed active equity funds for many years.‡

The outline of the paper is as follows. The first section presents the Grinold formula, the GK and CST prescriptions for active management with reference to the GK casino management rationale. The second section discusses the limitations for common practice of the overuse of the GK and CST prescriptions from an intuitive investment perspective. The third section provides a discussion of properties of index-relative mean-variance (MV) optimization and previous simulation studies relevant to our results. The fourth section presents our Monte Carlo simulation study that demonstrates that many applications associated with the fundamental law may be invalid and are often self-defeating. The fifth section provides a summary and conclusions.

GRINOLD'S FUNDAMENTAL LAW OF ACTIVE MANAGEMENT

Grinold (1989) theory is an approximate decomposition of the information ratio (IR) generally associated with active equity optimized portfolio management. Grinold shows that the MV optimization of a budget only or inequality unconstrained residual return investment strategy is approximately proportional to the product of the square root of the breadth (BR) and the information correlation (IC).§ Mathematically,

$$IR \cong IC * \sqrt{BR}$$

where IR = information ratio = (alpha) / (residual or active risk)

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* To be clear, we note that the term "estimation error" in this manuscript refers to Monte Carlo simulation experiments that measure how estimates of optimization parameters impact out-of-sample investment performance, and not, as in Zhou (2008) or Kritzman (2010), to refer to statistical estimation issues.

† Recently Allen et al (2019) and Kritzman (2006) have questioned the importance of estimation error in MV optimization. Michaud et al (forthcoming) and Michaud (forthcoming) demonstrate that estimation error is alive and very well when carefully parsed.

‡ We have recently become aware that the dimensionality issue of MV portfolio optimization relative to equal weighting was discussed in Bawa et al (1979, p. 129) in a somewhat different context taken from Stephen Brown's unpublished PhD dissertation.

§ The detailed derivation is given in GK, Ch. 6, and Technical Appendix.

IC = information correlation (ex ante, ex post return correlation)
BR = breadth or number of independent sources of information.

The formula teaches that successful active management depends on both the information level of the forecasts and the breadth associated with the optimization strategy. However, GK and CST go further. They apply the Grinold formula to assert that only a modest amount of information (IC) is necessary to enhance investment performance simply by increasing the number of assets in the optimization universe, the number of forecast factors, more frequent trading and reducing optimization constraints.

GK use a casino roulette game to rationalize applications of the Grinold formula to asset management in practice.* The probability or IC level of a winning play (for the casino) of the roulette game is small but more plays (breadth) lead to the likelihood of more wealth. However, there are important differences between the play of a roulette game in a casino and the play of an investment game in practice. In the casino context, probability of a winning play or IC is known, positive, and constant. In an investment game, the probability level is unstable and may often be negatively related to return. In the context of estimation error, increasing the number of plays of an investment game may often be undesirable. While entertaining, the casino game framework for rationalizing applications of the Grinold formula to actual investment practice is invalid.

DISCUSSION OF GK AND CST PRESCRIPTIONS

GK and CST propose four principles of optimized portfolio design for enhanced investment value in an index-relative MV optimization framework. In this section, we discuss from an intuitive perspective common practices that may often be negatively impacted by application of GK and CST prescriptions.

Large Optimization Universe

GK is often used to rationalize large stock universes in an optimized investment strategy. While theoretically, adding more assets may add to breadth, all other things the same, it may also result in less predictable securities and an overall reduction of average IC.

The issue can be framed in the context of an analyst suddenly asked to cover twice as many stocks. Given limitations of time and resources, it is highly unlikely that the analyst's average IC is the same for the expanded set of stocks. Similarly, analysts and managers attempt to specialize in areas of the market or investment strategies considered appropriate for the securities they cover. GK are well aware that their prescription for adding more securities in order to increase breadth is conditional on the skill level being maintained. However, average IC and optimization universe size are often negatively correlated in applications. Naively expanding the size of an optimization universe can often be self-defeating.

Multiple Factor Models

* The casino roulette game framework is illustrative of the assumptions in GK (1995, 1999, Ch. 6. App.)

Large stock universe optimizations typically use indices such as the S&P500, Russell 1000 or even a global stock index as benchmarks. In this case, individual analysis of each stock is generally infeasible and analysts typically rely on factor valuation frameworks for forecasting alpha. For example, stock rankings or valuations may be based in part on an earnings yield factor.[*] As GK note, if earnings yield is the only factor for ranking stocks, there is only one independent source of information and breadth equals one regardless of the number of securities involved.

In the Grinold formula, the IR increases with the number of independent positive significant factors in the multiple valuation forecast model. However, in practice, asset valuation factors are often highly correlated and may often be statistically insignificant, providing dubious out-of-sample forecast value.[†] Finding factors that are reasonably uncorrelated and significantly positive relative to ex post return is no simple task.

Factors are often chosen from a small number of categories considered relatively uncorrelated and positively related to return such as value, momentum, quality, dividends, and discounted cash flow.[‡] The breadth of multiple valuation models may often be very limited independent of the size of the optimization universe.[§,**] As in adding stocks to an optimization universe, adding factors at some point may include increasingly unreliable factors that are likely to reduce, not increase, the average IC of an investment strategy.

Michaud (1990) provides a simple illustration of the impact of adding factors to a multiple valuation model. While adding investment significant factors related to return can be additive to IC, it can also be detrimental in practice. There is no free lunch. Adding factors can as easily reduce as well as enhance investment value, and the number of factors that can be added while maintaining a desirable total IC is generally limited in practice.

Invest Often

GK recommend increasing trading period frequency or "plays" of the investment game to increase the BR, and thus the IR of a MV optimized portfolio. The Grinold formula assumes trading decision period independence and constant IC level. However, almost all investment strategies have natural limits on trading frequency.[††] For example, an asset manager trading on book or earnings to price will have significant limitations increasing trading frequency smaller

[*] Some standard methods for converting rankings to a ratio scale to input to a portfolio optimizer include Farrell (1983) and references.

[†] There is a practical limit to the number of independent investment significant factors even in many commercial risk models, often far less than ten.

[‡] Standard methods such as principal component analysis for finding orthogonal risk factors are seldom also reliably related to return over independent periods.

[§] See e.g., Michaud (1999).

[**] While principal component or factor analysis procedures for identifying orthogonal factors in a data set may be used, most studies find no more than five to ten investment significant identifiable factors that are also useful for investment practice.

[††] Special cases may include proprietary trading desk strategies where the information level is maintained at a reasonable level and trading costs are nearly non-existent. Other cases, such as high frequency and algorithmic trading are arguably not investment strategies but very low-level IC trading pattern recognition relative to highly sophisticated automated liquidity exchange intermediation.

than a month or quarter. Reducing the trading period below some limit will generally reduce effectiveness while increasing trading costs.

Fundamentally, trading frequency is limited by constraints on the investment process relative to investment style.* Deep value managers may often be reluctant to trade much more than once a year while growth stock managers may want to trade multiple times in a given year. Increased trading, to be effective, requires increasing the independence of the trading decision while maintaining the same level of skill. This will generally require increased resources, if feasible, all other things the same. The normal trading decision period should be sufficiently frequent, but not more so, in order to extract relatively independent reliable information for a given investment strategy and market conditions.

It is worth noting that the notion of normal trading period for an investment strategy does not imply strict calendar trading. Portfolio drift and market volatility relative to new optimal may require trading earlier or later than an investment strategy "normal" period. In addition, a manager may need to consider trading whenever new information is available or client objectives have changed. Portfolio monitoring relative to a normal trading period including estimation error is further discussed in Michaud et al (2012).

Remove Constraints

CST introduce the "transfer coefficient" (TC) that is intended to measure how information in individual securities and investment value in optimized portfolios may be reduced by the presence of constraints in the optimization. In common practice, MV optimized portfolios often include many linear constraints. Constraints added solely for marketing or cosmetic purposes may result in little, if any, investment value and may obstruct the deployment of useful information in risk-return estimates. But MV optimized portfolios are sensitive to estimation error in estimated inputs that often lead to unintuitive and impractical portfolios (Michaud 1989). Constraints are often imposed to manage instability, ambiguity, poor diversification characteristics, and enhanced out-of-sample performance.

More generally, inequality constraints are typically necessary in practice. Inequality constraints reflect the financial fact that even the largest financial institutions have economic shorting and leveraging limitations. Markowitz (2005) demonstrates the importance of practical linear inequality constraints in defining portfolio optimality for theoretical finance and the validity of many tools of practical investment management. Long-only constraints limit liability risk, a largely unmeasured factor in many risk models and often an institutional requirement. Regulatory considerations may often mandate the use of no-shorting inequality constraints. Performance benchmarks may often mandate index related sets of constraints for controlling and monitoring investment objectives.

TESTING GK AND CST PROPOSALS

* Trading costs and market volatility are additional considerations.

Investment managers often use a back test to demonstrate the likely value of a proposed investment strategy. In this procedure, a factor or strategy is evaluated on how it performed for historical data over some period. While the benefit of a back test may be practicality, no reliable prospective information is possible by definition. It is no less, and no more, than what happened over some historical period. Back tests are notorious for misleading investors, resulting in loss of wealth, careers, and dissolution of firms. Investors should be keenly aware of the serious limitations of any back test as evidence of the reliability of any factor relationship or investment strategy in practical applications.[*]

A simulation study is a far more reliable framework for testing the value of optimized investment strategies. Such a procedure evaluates the likely out-of-sample performance of an in-sample optimized portfolio for many realistic investment scenarios.

In the following sections, we explain the summary statistics used to evaluate the out-of-sample performance of investments from following the prescriptions of the fundamental law, describe the simulation test framework in detail, and discuss the results of our simulation experiment.

Portfolio Simulation Study Framework

Our study uses a framework similar to other well-known simulation studies for portfolio construction methods.[†] In this framework, a referee is assumed to know the true means, standard deviations, and correlations for a set of assets and consequently the true max Sharpe ratio (MSR) for an optimized portfolio of those assets. The players do not know the referee's true MV parameters. The players receive simulated returns based on the referee's parameters, so they can only observe the truth obscured by estimation error, as is true for all real-world investment managers. The players then compute optimal weights for their strategies and report the simulated MSR optimal portfolios for the referee to score. The referee determines the true Sharpe Ratios (SRs) for the simulated MSR optimal portfolios. The procedure is repeated many times for referee-simulated returns, and averages of true simulated optimal portfolio SRs computed for each player. In this way, the out-of-sample performance of each player's strategy can be compared, and the better strategy determined.

[*] Even long-term academic studies remain susceptible to unreliability in practice.
[†] For example, Jobson and Korkie (1981).

Prior MV Optimization Simulation Studies

Jobson and Korkie (JK) (1981) provide the classic study of the effect of estimation error on the out-of-sample investment value of budget only or unconstrained MV optimized portfolios for an optimization universe of 20 stocks.* In their study, the referee's truth is based on historically estimated MV inputs for twenty stocks. They Monte Carlo simulate returns assuming a multivariate normal distribution of five years of monthly return data. They find that the average of the true SRs, as measured by the referee, of simulated MSR optimal portfolios, was twenty-five percent of the true MSR of the referee's optimal portfolio. In addition, they show that equal weighting substantially outperforms the optimized portfolios.† They conclude that budget only MV optimization is not recommendable for practice.

Frost and Savarino (FS) (1988) perform a related simulation study for long-only MV optimized portfolios for a 200 stock universe. They find that sign and additional constraints often add investment value to the out-of-sample performance of MV optimized portfolios. Economically realistic constraints may often act like Bayesian priors focused on portfolio structure enforcing rules representing legitimate information not contained in the optimization inputs. Such restrictions can mitigate estimation error in risk-return estimates implicitly by forcing the simulations towards more likely optimal portfolios.

We note that the JK and FS studies contradict the theoretical results of CST for two different size stock universes. Our study confirms and generalizes their results conditional on optimization universe size.

SIMULATING ADDING BREADTH WHILE MAINTAINING INFORMATION LEVELS

In the standard interpretation of the Grinold formula, each spin of the roulette wheel adds one unit of breadth to the investment game. There is no uncertainty as to whether additional spins will continue to be advantageous for the house even when the odds are only slightly in its favor. In our simulations, the critical deviation from the GK roulette wheel framework is that estimation error of the probability of a win for the investment house is neither known nor constant. Our objective is to construct a Monte Carlo simulation in the context of estimation error where each randomly selected incremental asset has variable information but a constant expectation of adding one unit of breadth. Therefore, the simulated optimized portfolios will be affected by estimation error, but the average of the simulations will exhibit constant incremental breadth.

Simulation Methodology

* Note that the JK study applies equivalently to budget only constrained quadratic utility portfolio optimization, a framework widely used in financial theory such as the CAPM and for the development of many investment strategies.

† An equal weighted portfolio is a simple way to compare the optimality of budget only constrained optimized portfolios.

We begin with a sample of historical market return data* of 500 stocks that will be the basis for our simulations. This particular dataset is immaterial to our argument. What is essential is that the master dataset represents a realistic set of expected returns and full-rank covariance matrix for the largest sample size of the experiment.†

We propose a novel simulation framework that consists of random sampling without replacement of increasing size subsets of the 500 stocks of the referee's risk-return estimates from the master optimization universe. The averaging of the results of thousands of samplings without replacement from the given 500 stock universe with increasing size provides a solution to estimating the Grinold theoretical concept of increasing a unit of breadth for increasing number of assets. In this way, the functional form of average out-of-sample simulation performance can be compared to Grinold theory prediction of a monotonic increasing concave function of breadth.

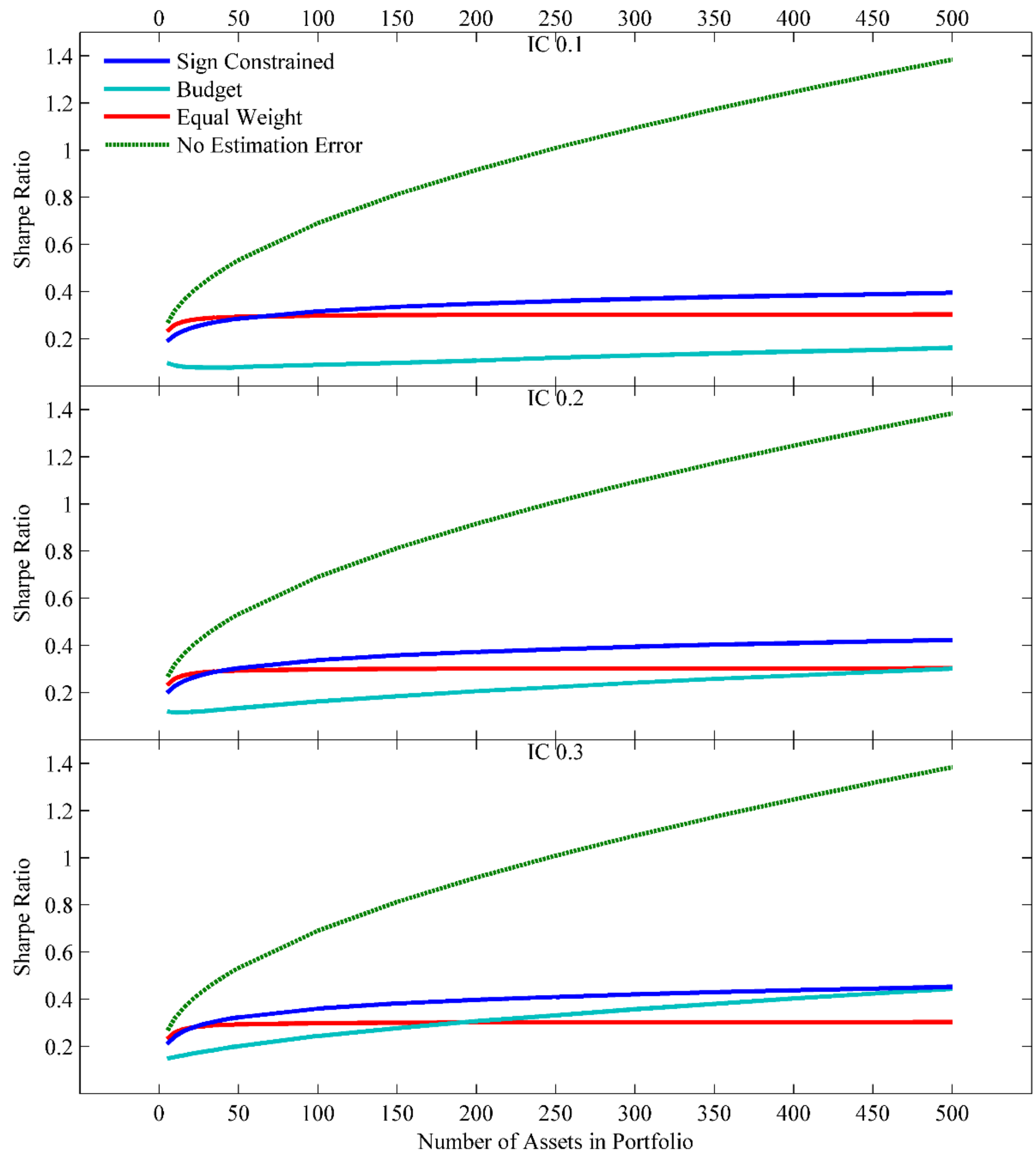
We Monte Carlo simulate returns assuming a multivariate normal distribution for the referee's mean and covariance matrix. Each simulation consists of sampling without replacement of the 500 stocks of increasing size to 500 assets. The referee's truth is computed by independently adding assets to the referee's expected return and covariance matrix. The problem of computing a sample covariance from simulated returns is avoided by assuming the referee's covariance. This assumption eliminates non-full-rank covariance estimation from simulated returns as a plausible explanation of our results.‡ It also means that our results represent a very generous upper bound of average out-of-sample performance for actual practice.

* We use a recent history of US market data (1994-2013) of publically available data to create our master asset list and corresponding mean and variance parameters. We selected all the assets from the largest 1000 in market capitalization with contiguous data from the period, excluding returns greater than 50% or less than -50% per month. We were able to find 544 stocks that met our criteria. Parallel experiments with shorter histories were also run to investigate if selection bias affects results, with no positive findings, so we present the twenty-year history here. Readers wishing to replicate our experiment can access our data at http://newfrontieradvisors.com/media/1657/estimation_error_and_the_fundamental_law_data.csv.

† A principal components decomposition of our referee's covariance matrix confirms that none of the independent dimensions of the system vanish. All of the eigenvectors are needed to replicate our forecast to reasonable precision. If some of the eigenvalues were vanishingly small, the practical answer to the question of breadth would be quite different from the mathematically rigorous one. However, the full covariance matrix of 500 assets in our dataset has a smallest eigenvalue of over 10 basis points, which is likely significant for most definitions of statistical significance. This would correspond to an annualized standard deviation of approximately 11%, which is substantial by most measures. The submatrices of smaller portfolios tend to have even greater values for the smallest eigenvalue. This line of reasoning confirms that the effective breadth of a sample of size N from our universe is identically N in a practical sense as well as the theoretical one.

‡ In particular, this assumption avoids the issues in Fan et al (2008).

Exhibit 1: Average Maximum Sharpe Ratios by Information Level, Referee Covariance



Note: Average SRs for three different portfolio construction methods and three different information coefficients for the equity optimization case, using the referee's covariance matrix. Target information coefficients are not precisely attained by the simulations; the sample sizes in each IC category were chosen to best approximate the target ICs. This experiment was run on many simulations of up to 500 U. S. stocks which had at least 20 years of contiguous monthly price data ending in December 2013.

Budget Constrained, Long Only, and Equal Weight

Exhibit 1 reports the results of our simulation studies. It consists of three panels of simulation results for 0.1, 0.2, and 0.3 IC levels of estimation error from optimization universes from five to 500 assets. Each value presented on the graph is averaged from 16,000 samplings without replacement optimizations. The three graphed series in each panel show the out-of-sample average SRs resulting from three different optimization methods. The "budget constrained" series displays the out-of-sample averages of the true SRs for the simulated budget-only constrained MSR portfolios, the "equal weight" series displays the average true SRs of equal weighted portfolios, and the "constrained" series reflects the out-of-sample averages of the true SRs of simulated long-only MSR portfolios. The fourth line in green reflects the average SRs over the simulations for budget only constrained MV optimization using the referee's returns, risks, and correlations, i. e. with no estimation error, as a function of optimization universe size. Although not precisely proportional to the square root of universe size, the green curve is notably much closer to the predictions of the Fundamental law than the others which include some estimation error.

Our simulations confirm and generalize the simulation study results in JK and FS and provide a rationale for the empirical results in deMiguel et al (2007). In particular, equal weight is far superior to budget constrained optimization, as shown in JK, for optimization universes of modest size, such as asset allocation studies with generally less than 50 securities. For larger optimization universe, the results are consistent with FS, where long only dominates budget constrained and equal weight. Our results provide a single consistent framework for summarizing and extending the classic results in historical studies on estimation error.

The three levels of the Grinold assumed IC: 0.10, 0.20, and 0.30 is computed by varying the number of periods of simulated returns for each size universe. Because of the Monte Carlo nature of our experiment, the average realized ICs for each sample size are close but not exactly equal to target.[*] The observation sizes for each target IC were determined by calibrating the largest portfolio size (500) for the experiment. While IC levels greater than 0.10 are not formally applicable to predictions from the Grinold formula, our simulations transcend assumptions in the theory and may have important teachings in other investment applications. While each of the reports for stock subsets without replacement will necessarily reflect randomness of adding stocks, averaging over 16,000 such simulations should represent a very reliable estimate of additive breadth for a realistic data set of historical returns. Our results should be reasonably representative for similar datasets of practical interest.

In the case of IC equal to 0.30, the out-of-sample budget constrained performance nearly attains the level of the constrained case for the largest sample size of 500 assets. However, these experiments avoid any consideration of financial frictions or costs that would limit the investment value of large universe optimized portfolios.[†] Our assumption of an error free covariance matrix further upward biases our simulations. Our results should provide convincing evidence of the

[*] Results are available on request.

[†] See for example Grossman and Stiglitz (1988).

limitations of ignoring estimation error and the need for inequality constraints in practice for optimal portfolio optimization design.

Further Discussion

Our deliberate optimism on how additive breadth is modeled when increasing the size of the optimization universe in the simulations has important implications. All of the assets in the simulation universe are assumed to have some investment value. Consequently, an investor is little harmed by putting portfolio weight on a "wrong" asset. In the real world, constraints often limit the harm caused by misinformation. In a truly chaotic world with a lot of estimation error and bias, the equal weighted portfolio, which uses no "wrong" information to distinguish among assets, can be hard to beat, for small optimization universes such as in asset allocation strategies.

The consistent slow rising level of budget constrained simulated optimized portfolio average true SRs as universe size increases is a necessary artifact of our simulation framework. This is because, by design, our simulations assume a consistent level on average of realized IC regardless of universe size. In practice, many investment strategies have an optimal universe size. Beyond some point, increasing universe size is likely to be self-defeating in practice.

SUMMARY AND CONCLUSIONS

Our narrative does not contradict the simple intuition that investment performance is a function of skill and breadth. It is always true that it is better to have more reliable information (IC) and more additional investment opportunities to apply it (BR). However, the only truly reliable message is that it may take considerably more than a modest amount of skill to win the investment game independently of whether it is deployed frequently or across how many stocks. As we have demonstrated under realistic assumptions, naively adding assets or factors or removing constraints may often be self-defeating. Even under the highly idealized conditions of our simulation study where BR can be additively applied while holding IC level constant, the results in Exhibit 1 dramatically contradict the predictions of Grinold theory.

The results in Exhibit 1 generalize and extend the classic JK and FS simulation studies and help rationalize the empirical "1/N" results in deMiguel et al (2007). Our simulation framework enables characterization of the theoretical notion of Grinold "breadth." From our simulation study, and from simple investment considerations, estimation error affects many of the results in the large body of published papers associated with applications of Grinold theory.

Our results have important implications for contemporary investment practice. For more than twenty-five years, Grinold theory applications have often been considered the canon of professionally managed quantitative equity funds. Many rationales for investing in hedge funds, long short, absolute return, high frequency trading, alternatives, and minimally constrained strategies may be impacted.

The necessary conditions for winning the investment game remain the fundamental principles of reliable long-term asset management: 1) investment significant information and high quality investible assets relevant to a given size optimization universe; 2) economically meaningful constraints; and 3) properly implemented estimation error sensitive portfolio optimization technology.

The fundamental root of the failure of applications of the theoretical Grinold formula is one of many examples of the ubiquitous fallacy in many areas of social science of regarding inference from in-sample statistics and fixed probability models as the full measure of uncertainty.*

* Knight (1921) and Weisberg (2014).

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