NEW FRONTIER

Resampled Efficiency™ for Financial Planning and Portfolio Return Forecasting

by

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Abstract

An examination of the effect of forecast certainty level indicate that the enormous effort focused on input estimation by many managers and institutions without Resampled Efficient Optimization is misplaced and likely to be ineffective.

We present new evidence that Resampled Efficient Frontier™ (REF) portfolio optimization is a dominant technology relative to classical mean-variance (MV) optimization. MV optimization typically creates significant overestimates of future portfolio return whatever the level of information in optimization inputs. This property makes MV efficiency unsuitable for financial planning, portfolio return forecasting and many other uses of optimized portfolios in investment management. In contrast, good inputs are likely to produce realistic optimized portfolio return forecasts with our resampled optimization methods.[1] Coupled with earlier proofs of dominance in out-of-sample performance, these signature results demonstrate the power and importance of REF optimization for many practical applications in asset management.

Objectives

Resampled Efficient Frontier or Resampled Efficiency portfolio optimization is a patented procedure for improving the investment performance of optimized portfolios.[2] Resampled Efficiency controls estimation error by allowing the user to assign a Forecast Certainty™ level that is consistent with the presumed level of information in the optimization inputs. This report examines the effect of Forecast Certainty level misestimation on investment performance.[3] We show that Resampled Efficiency with any Forecast Certainty level is preferable to classical mean-variance (MV) efficiency.

Stein estimation methods, often proposed to reduce estimation error for portfolio optimization, were included in our studies.[4] Even when Stein estimation is used, MV efficiency exhibited critical limitations for efficient frontier forecasting, cash flow financial planning, and out-of-sample performance. On the other hand, Stein estimation and Resampled Efficiency provided very useful estimates of actual out-of-sample portfolio return. Properly understood, these results show that REF optimization is a necessary condition for useful portfolio return forecasting and improving out-of-sample performance. These results also indicate that the enormous effort focused on input estimation by many managers and institutions without Resampled Efficiency optimization is misplaced and likely to be ineffective.

Background

Few argue the importance of portfolio diversification for asset management. The fundamental contribution of Markowitz (1952, 1959) mean-variance (MV) portfolio efficiency in practice is a procedure for efficiently diversifying risk without reducing expected return. Unfortunately the theoretical benefits of MV efficient diversification

[1] Some imitations of our resampled optimization methods do not have such properties.
[2] Described in Michaud (1998, Chs. 6, 7), Resampled Efficient Frontier portfolio optimization was co-invented by Richard Michaud and Robert Michaud and is a U.S. patented procedure, #6,003,018, December 1999, patent pending worldwide. New Frontier Advisors, LLC (NFA) has exclusive worldwide licensing rights.
[3] To facilitate the user experience, NFA has calibrated ten Forecast Certainty levels. These range from one or very uncertain, to ten or very certain. The level of diversification of the resampled optimal portfolios decreases as the level of certainty increases. At very low levels of certainty the optimized portfolios are much diversified. At high levels of certainty the optimized portfolios are similar to classical optimal portfolios. See Michaud and Michaud (2003) for discussion of some associated issues.
[4] See Michaud (2003a) for further discussion. A number of updates of Resampled Efficient Frontier technology are also available at www.newfrontieradvisors.com.

are much less useful in practice. MV optimized portfolios are estimation error insensitive and typically exhibit poor out-of-sample investment performance. Tests have shown that an investor is often better off equal or benchmark weighting than using a MV optimizer.

The essential problem of MV optimization is that input information is treated as certain. But investors are never 100% certain of their information. An investment useful portfolio optimization process requires using investment information in a statistical, rather than certain, sense.

Roll (1979) was the first to note that Markowitz mean-variance (MV) portfolio optimization requires a statistical approach. Jobson and Korkie (1980, 1981) were the first to show that MV optimization results in substantial upward biases of true portfolio optimality and to indicate the potential benefits of Stein return estimation. Michaud (1989) described the Jobson and Korkie MV optimization bias as an "error maximization process." Michaud (1998) proposed Resampled Efficiency optimization for controlling the impact of estimation error in optimized portfolios.
REF optimization requires the user to assume a Forecast Certainty level to reflect the assumed information level in the inputs. In the following we examine the impact of misestimation of Forecast Certainty level. We also study the effect of Stein estimation on out-of-sample optimized portfolio performance.

Forecast Certainty Level Tests

Return generating process: A simulation test of out-of-sample optimized portfolio investment performance requires a definition of the true return generating process.[5] We use the means, standard deviations, and correlations of the eight asset class data in Michaud (1998, Ch. 2) to define true return.

Information certainty: Sixty-months of simulated returns are used to compute simulated efficient frontiers from the true optimization inputs. While sixty-months of simulated returns is formally similar to observing sixty-months of historical return data, because the process is stationary by definition, the information certainty of the simulated returns is significantly higher than for sixty-months of historical return data.

Forecast Certainty levels: We examine three levels: 1, 3, and 5. Level 3 is comparable to the information certainty in the simulated returns. The three levels bracket the range of investment consequences: too little certainty (1), roughly the correct level of certainty (3), and too much certainty (5).

Study framework: There are six cases or exhibits at the end of this article. Exhibits 1-3 and 4-6 vary by the three Forecast Certainty levels. Exhibits 1-3 use traditional input estimation while Exhibits 4-6 use Stein estimation. Each exhibit displays five efficient frontiers described below. We use 500 simulations to compute in- and out-of-sample performance.

[5] Unlike historical back tests which have little reliable information, simulation tests may reliably evaluate the investment value of various procedures on portfolio performance. See Michaud (2001).

1. The true efficient frontier (black) is computed from the true optimization inputs.
2. The out-of-sample average classical efficient frontier (red) represents the out-of-sample average performance of 500 simulated classical optimized efficient portfolios.
3. The out-of-sample average Resampled Efficient Frontier (blue) represents the true out-of-sample average performance of 500 simulated REF portfolios.
4. The in-sample average classical efficient frontier (pink) represents the in-sample average of 500 simulated classical efficient frontiers.
5. The in-sample average Resampled Efficient frontier (light blue) represents the in-sample average of 500 simulated REF portfolios.

Stein estimation: Stein estimators are shrinkage operators that typically improve the forecast value of parameters estimated from historical return data. Stein estimators have been proposed as a way to reduce the effect of estimation error on portfolio optimization.[6] Exhibits 4-6 include Stein estimation of optimization inputs.[7]

Results

The red and blue curves in the six exhibits compare average out-of-sample investment performance of classical versus resampled optimization. The results show that Resampled Efficiency optimization provides roughly the same level of average return with less risk.[8] This is a standard rationale for preferring resampled to classical efficiency.

Exhibits 1-3 and 4-6 compare the effect of Forecast Certainty level estimates. Unsurprisingly, a good estimate of the correct level of information certainty is desirable; relative to classical efficiency, too little certainty leads to less willingness to take useful risk while too much certainty leads to less improvement in out-of-sample average return. However, misestimating certainty appears to do little harm; any uncertainty level leads to more desirable performance than the 100% certainty implicitly required by classical efficiency.

Compare the average in-sample classical (pink), resampled (light blue), and true (black) efficient frontiers in Exhibits 1-3. Note how poorly on average the observed classical efficient frontier (pink) estimates the true return distribution (black curve). This is the error maximization effect first demonstrated by Jobson and Korkie. While less extreme, in-sample average Resampled Efficient Frontier optimization (light blue) is also a poor estimate of the true return distribution.

Now compare the in-sample classical (pink) and resampled (light blue) efficient frontiers to their out-of-sample counterparts in Exhibits 1-3. The in-sample classical efficient frontiers are extremely poor out-of-sample estimates of average return; the resampled in-sample frontiers are less upward biased but still poor estimates of average performance. From

[6] See Jobson and Korkie (1981) and Michaud (1989, 1998 Ch. 8). NFA's asset allocation software suite includes Stein return estimation.

[7] The two Stein estimation procedures are James-Stein-Efron-Morris for return and Ledoit for covariance estimation. See Michaud (1998, Ch. 8) for further information and references.

[8] See Michaud (1998, Ch.6) for further details.

Exhibit 3 relative to 1 or 2, there is some evidence that is better to underestimate, than overestimate, Forecast Certainty level. It is worth noting that risk estimation is relatively reliable.

Finally, compare the effect of Stein estimation on the results in Exhibits 4-6. Relative to out-of-sample average return, the upward bias of in-sample classical frontiers is still substantial though much diminished. However, the Stein estimated in-sample REF optimizations are not very different from their out-of-sample counterparts. While in-sample classical efficiency with Stein estimation still gives a false view, in-sample Resampled Efficiency optimization with Stein estimation gives a very useful even relatively realistic view of actual return on average.

Cash Flow Financial Planning and Other Applications

Cash flow financial planning and efficient frontier forecasting are important activities for many asset managers, strategists, consultants, investment advisors, and financial planners. In particular, the investment value of multiperiod cash flow planning is contingent on the realism of forecasts of portfolio risks and returns.[9] With or without Stein estimation, our results show that classical efficiency is highly upward biased and not useful for cash flow planning or efficient frontier forecasting. On the other hand, Resampled Efficiency optimization can be very useful for these and other important applications using optimized portfolios.

Stein Estimation Discussion

The combination of Stein and Resampled Efficiency optimization results imply that the inputs were excellent optimization estimates. However, even with excellent inputs, classical efficiency provides poor out-of-sample estimates of portfolio return, not to mention inferior out-of-sample performance. Consequently, classical efficiency should be avoided for cash flow financial planning and efficient frontier forecasting.

Analysts and strategists often use non-Stein methods to estimate optimization inputs. The effort of analysts, strategists, and financial institutions to formulate useful forecasts of asset return will often be compromised by the out-of-sample limitations of MV efficiency. On the other hand, the better the input estimates the better Resampled Efficiency is likely to perform.

Open Issues

Simulation studies provide very useful evidence in cases where there are no analytical solutions available. However, such solutions are necessarily case dependent and excessive generalizations may be unreliable. Nevertheless, our results should provide much comfort to those who use Resampled Efficiency for efficient frontier forecasting, asset allocation, equity portfolio optimization, and financial planning while providing serious warnings to those still using traditional optimization methods.

[9] For a recent review of portfolio choice issues for financial planning see Michaud (2003b).

Summary and Conclusion

Our results reconfirm that REF optimization is a dominant technology relative to classical MV optimization under a broad set of assumptions. Regardless of Forecast Certainty level assumed, Resampled Efficiency optimization provides the investor with roughly the same range of average return without needing to assume as much risk. The value of conservative estimates of Forecast Certainty for ignoring misleading perceptions of high-risk portfolios may need to be balanced relative to the lost opportunities of ignoring useful risk.

Our study illustrates the enormous upward bias in expected return for classical efficient frontiers relative to out-of-sample investment performance with or without Stein estimation. In contrast, Stein estimated Resampled Efficiency optimization provides a very useful estimate of average out-of-sample investment performance. Taken together, these results imply that classical MV efficiency provides false estimates of portfolio return even with excellent inputs and therefore cannot be recommended for cash flow financial planning or efficient frontier forecasting. In contrast, Resampled Efficiency uses useful optimization inputs well. Our results demonstrate that Resampled Efficiency is a necessary condition for useful out-of-sample forecasts of portfolio return. Since cash flow financial planning and efficient frontier forecasting using classical optimization and estimation methods is widespread in the investment community, many are well advised to review their procedures in the light of these important results.

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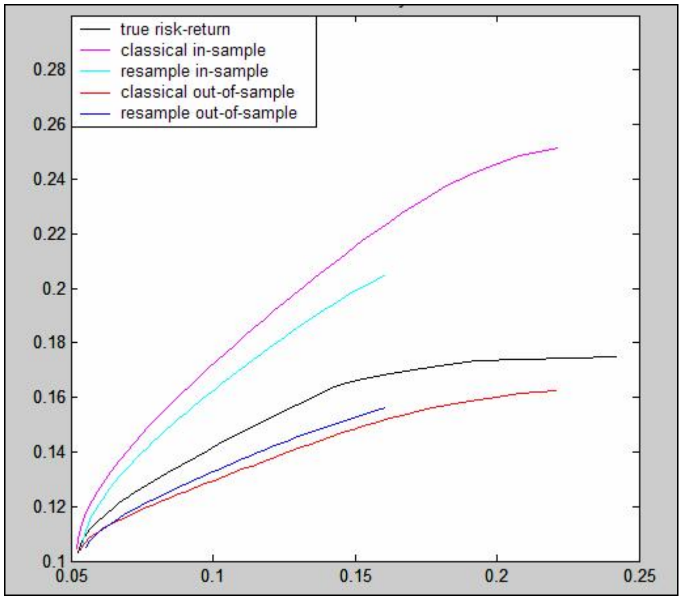

Exhibit 1: Forecast Certainty = 1

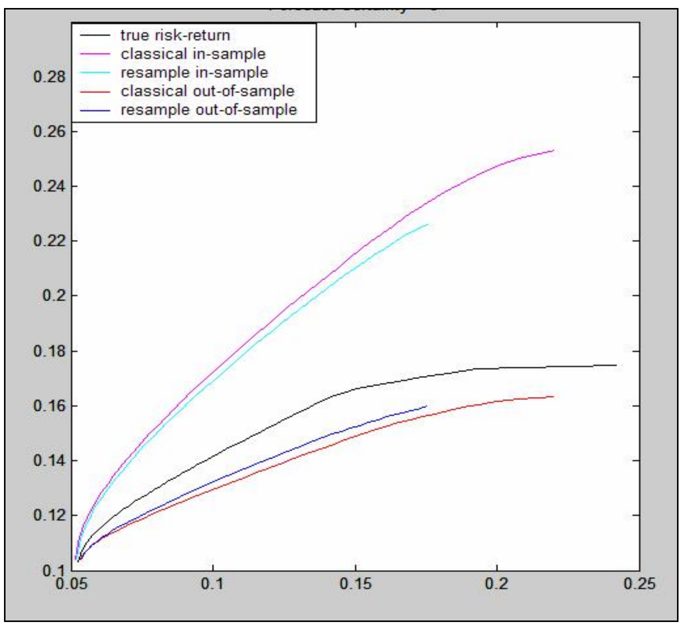

Exhibit 2: Forecast Certainty = 3

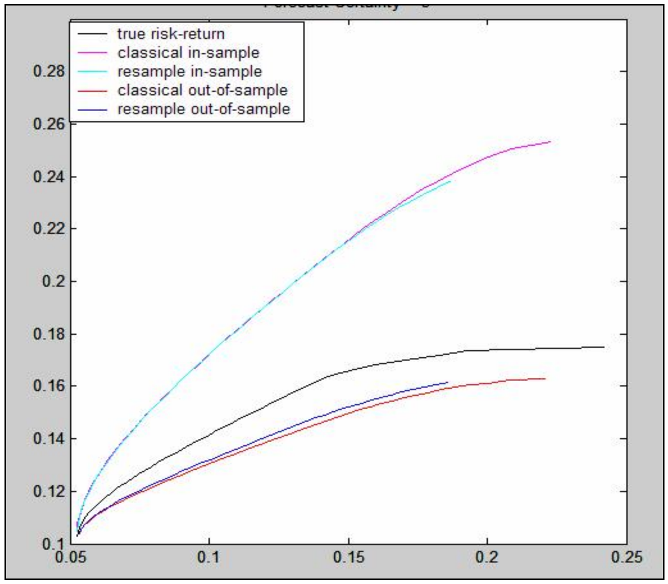

Exhibit 3: : Forecast Certainty = 5

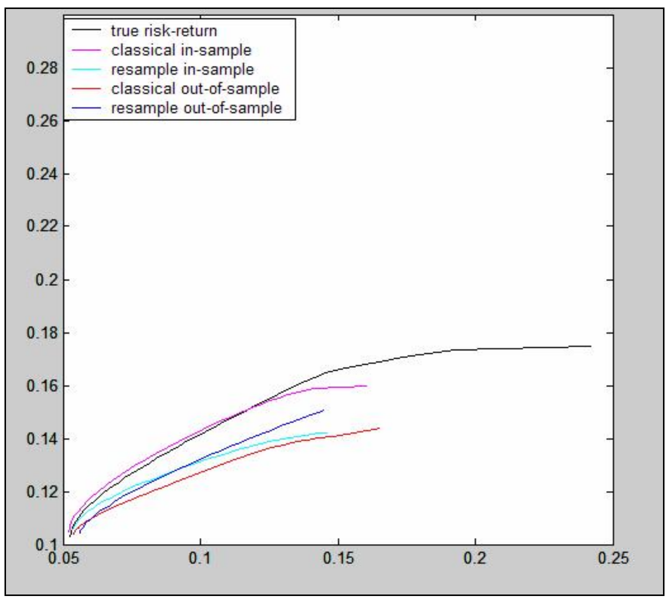

Exhibit 4: Stein Estimation, Forecast Certainty = 1

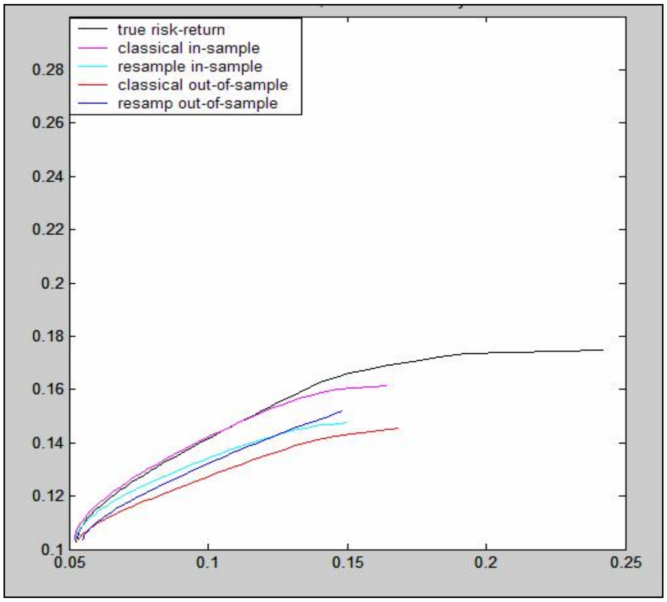

Exhibit 5: Stein Estimation, Forecast Certainty = 3

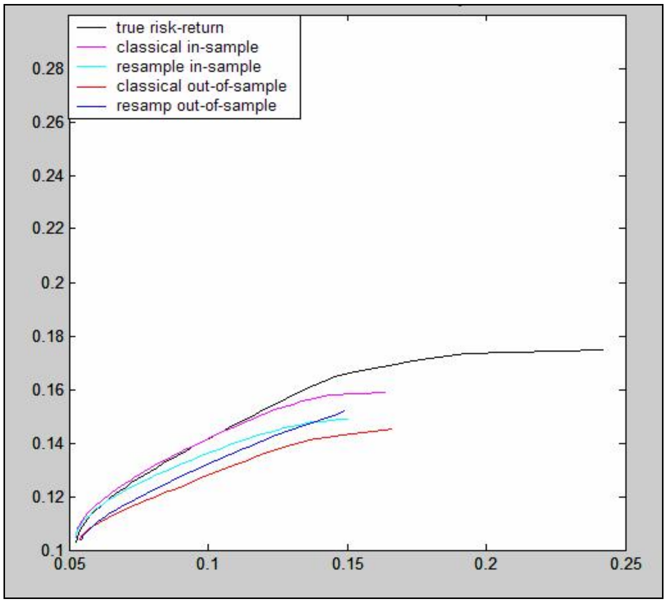

Exhibit 6: Stein Estimation, Forecast Certainty = 5