NEW FRONTIER
ADVISORS, LLC

Resampled Efficiency™

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The proper purpose of investment advice is to improve a client's portfolio in terms of maximizing return for an appropriate level of risk. Asset management techniques for optimizing the investment value of forecasts of return and risk have been available for fifty years. Markowitz provides the classic definition of optimality: a portfolio is risk-return efficient if no other portfolio has higher expected return for a given level of risk or less risk for a given level of expected return.[1] In classical optimization, the set of all portfolios that are risk-return efficient are said to form the efficient frontier. Yet, despite its conceptual brilliance, classical optimization doesn't work.

The central problem lies in how classical optimization works with inputs. To illustrate, suppose you forecast a 10% return and a 20% standard deviation for the S&P500 index. This could easily turn out to be a return of 13% or 8% with a standard deviation of 25% or 15%. There is, of course, uncertainty in the forecasts of other assets as well. While estimate uncertainty may be relatively easy to manage with one or two assets, it becomes all but impossible when the portfolio has five or more assets or funds. Optimization technology exacerbates the problem. Even small changes in optimization inputs often lead to large changes in optimized portfolios, especially as computers interpret each input to fifteen decimal places. This degree of sensitivity to the inputs only works if the estimates can be trusted to be exact forecasts. Since investment information is never known with certainty, the resulting instability of the procedure leaves users with little confidence in the results. Advisors, aware of this problem, often include many constraints in order to stabilize the optimization, but manual stabilization can easily negate the value of the original optimization.

Statistical evidence confirms advisors' intuition concerning the uselessness of classical optimization. The evidence came more than twenty years ago, in a series of papers authored by two financial economists, J.D. Jobson and Bob Korkie, of the University of Alberta. They showed mathematically and statistically that optimizers have little, if any, investment value and that portfolios that equal weight all of the assets are often far superior to optimized portfolios.[2] The key to understanding the limitations of portfolio optimizers in practice provided by Jobson and Korkie's paper concurs with advisors' intuition. Classical optimizers assume 100% certainty in the information. But investment information is inherently uncertain. Optimizers tend to "error-maximize" investment information creating portfolios that are far too specific to a given set of inputs and consequently have little investment value.[3] So, can anything be salvaged from the concept of optimality and the efficient frontier?

Resampled Efficiency™, invented and patented by Richard Michaud and Robert Michaud, provides the solution.[4] The method is based on resampling optimization inputs. Resampling is a Monte Carlo simulation procedure to create alternative optimization inputs that are consistent with the uncertainty in all forecasts. The resampling process uses the advisors'

[1] Harry Markowitz, 1952. "Portfolio Selection." *Journal of Finance* 7(1): 47-62.
[2] J.D. Jobson and Bob Korkie, 1981. "Putting Markowitz Theory to Work." *Journal of Portfolio Management* 7(4): 70-74.
[3] R. Michaud, 1989. "The Markowitz Optimization Enigma: Are Optimized Portfolios Optimal?" *Financial Analysts Journal*.
[4] U.S. Patent # 6,003,018, December 1999.

forecasts to indicate the many ways capital markets and assets may behave in the investment period. This idea is very similar to observing the behavior of a fair coin when tossed ten times. You expect to see five heads in ten tosses on average. But actual tosses of the coin may result in two or nine heads even when the coin is fair. Simulation can show you how many ways the number of heads can occur and similarly the many ways assets and capital markets can behave. There are many alternative likely optimization inputs, efficient frontiers, and optimized portfolios consistent with forecast information. These alternatives are critically important for defining investment meaningful optimized portfolios. Resampled Efficiency simulates the possible results of different asset weights in a portfolio. Return, risk, and the relationships between assets are all treated as uncertain forecasts.

But Resampled Efficiency is not simply a way of creating consistent alternative efficient frontiers and optimized portfolios. Resampled Efficiency is an averaging process that distills all the alternative efficient frontiers into a new efficient frontier and set of optimized portfolios. Resampled efficient portfolios are optimal with respect to the many ways assets and capital markets can perform in the investment period and still be consistent with the forecasts.

Resampled Efficiency is very simply a better way to use investment information. It also has many investment attractive properties. Because Resampled Efficiency is an averaging process, it is very stable. Small changes in the inputs are generally associated with only small changes in the optimized portfolios. Resampled optimal portfolios are typically very investment intuitive without the need for constraints. This is because the uncertainty in the forecast information, which is ignored with classical optimization, is considered when defining the resampled optimized portfolio. Most importantly, Resampled Efficiency can be shown to improve investment performance, on average.

The historical risk-return data and optimized portfolio weights for the six indicated asset classes in the table below provide a simple comparison of the classical and resampled optimized processes near the middle of the frontier. Note that, in this case, the classical portfolio has no allocation to small cap stocks. This is because the risk-return estimates are used literally in classical optimization, and small cap stocks have inferior estimates of return relative to large caps. This also means that the classical portfolio is extremely vulnerable to a disappointing large cap stock return. In contrast, Resampled Efficiency includes a prominent component of small cap stocks. This is because Resampled Efficiency is sensitive to uncertainty and uses investment information in a more robust manner. While the two portfolios are comparable, the Resampled Efficient portfolio is better diversified and less risky.

Classical vs. Resampled Optimal Portfolios[5]

Assets	Exp. Retn (%)	Std. Dev (%)	Classical (%)	Resampled (%)
T-bills	4.6	0.4	0	6
Interm. Govt.	7.1	4.3	36	18
LT Corp	8.3	6.4	0	12
LC Stock	17.7	13.4	64	54
SC Stock	15.7	17.4	0	10
Intern Stock	8.3	17.1	0	0

Markowitz, the father of classical optimization, decided to test Resampled Efficiency in 2003. He, along with colleague Nilufer Usmen, tested classically optimized portfolios against resampled efficient portfolios in a refereed simulation test. To Markowitz's surprise, the resampled efficient portfolios performed better in all thirty tests, even when the classical optimizer used forecasts that were closer to the truth. The tests showed that Resampled Efficiency promises less and achieves more.[6] Incorporating uncertainty greatly enhances portfolio optimization.

Resampled Efficiency is a new more practical definition of portfolio optimality. It also includes a rigorous, patented tool for portfolio monitoring and rebalancing. Choosing when to adjust your portfolio has always been difficult. Judging superficial similarities between portfolios is typically an unreliable guide. Each portfolio on the Resampled Efficient Frontier is the result of averaging a number of statistically equivalent efficient portfolios. This creates what may be described as a fuzzy set of statistically equivalent portfolios associated with each portfolio on the Resampled Efficient Frontier. For a given Resampled Efficient Frontier, the fuzzy region of associated portfolios, in its simplest form, can be visualized as a set of points filling the inside of an American football with the resampled efficient portfolio in the center.

Now think about comparing your current portfolio to a portfolio on the Resampled Efficient Frontier. Is your portfolio inside the football and close to the center or far from the center or even outside the football? If it's far from the center, it's not statistically similar, and you may want to consider rebalancing it. On the other hand, if your portfolio is close to the center, you may not need to rebalance the portfolio at all because it's statistically similar. This is the patented Resampled Efficiency Rebalancing Test, the first statistically rigorous portfolio rebalancing rule available to the investment community, which is used to compute a need-to-trade probability. Using it ensures that unnecessary trades, with the attached trading costs, are avoided.

Portfolios developed using the patented Resampled Efficiency process are more stable, more precise, and more diverse. The Rebalancing Test reduces unnecessary trades. Most importantly, Resampled Efficiency expects and plans for uncertainty. New Frontier Advisors employs Resampled Efficiency in all of its services.

[5] The first two columns in the table display the annualized means and standard deviations of monthly data for six asset classes over 10 years. The last two columns display the classical and resampled optimal portfolio weights with equal annualized standard deviations of 9% in the middle of their respective frontiers.

[6] H. Markowitz & N. Usmen, 2003. "Resampled Frontiers vs. Diffuse Bayes: An Experiment." *Journal of Investment Management* Fourth Quarter: 1(4).