NEW FRONTIER

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Enhanced Return ▪ Effective Diversification ▪ Managed Risk

Dr. Richard O. Michaud

President and CIO,
New Frontier

Dr. Michaud earned a Ph.D. in Mathematics from Boston University and has taught investment management at Columbia University. He is the co-holder of 4 U.S. patents and author of *Efficient Asset Management* and many professional and academic articles.

Dear Friends of New Frontier,

I would like to share an important article by Dr. David Esch, our Director of Research. David succinctly reviews some fundamental innovations he has developed and introduced in New Frontier's asset management software. Each innovation represents the state-of-the-art in contemporary investment management technology. All these procedures are being used in our own asset management process.

I'd also like to note that the interview that I conducted with Dr. Harry Markowitz at last spring's *Journal Of Investment Management* conference will be published in that distinguished journal in the last quarter of this year. Finance has made many important developments during his lifetime, thanks in great part to his accomplishments. I hope you enjoy reading the interview as much as I enjoyed conducting it.

Members of our research team are racking up the frequent flyer miles traveling around the country to attend conferences and to bring the New Frontier story to financial advisors and investment committees around the country. We hope to connect with you at one of these events.

--Dick

Several Quantitative Innovations That May Improve a Portfolio's Expected Performance

by Dr. David Esch

Michaud Optimization is the cornerstone of the New Frontier investment process. Its basic principle is including the uncertainty of investment information by averaging together many mean-variance efficient frontiers based on various (resampled) capital market assumptions. Michaud optimization provides a solid foundation of reliability across multiple scenarios, but there is always room for improvement. This article discusses some important adjustments which can increase performance on top of the normal advantages of Michaud optimization. The advantages from these adjustments are additive, so together they add substantial value by increasing expected return or reducing risk. Among the fertile areas of innovation and research are (1) how the resampled assumptions are generated, and (2) which portfolios are chosen from each frontier to create the final result. Recent research has resulted in some best practices , which are now important parts of our investment process. Below, we outline

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Dr. David Esch

Dr. Esch earned his PhD in Statistics from Harvard University in 2004. He has specialized knowledge in mathematical statistics, numerical analysis and computation, Bayesian statistics, and econometrics. He is the author of the article "Non Normality Facts and Fallacies," (*Journal of Investment Management* 1st quarter 2010), selected as one of the best *JOIM* papers of 2010. A native of Boston, he attended Harvard College before obtaining a master's degree in mathematics from Boston University.

three of these best practices and discuss some of the gains that can be expected when using them. The three methods are (1) the missing data procedure, (2) the t distribution for resampling, and (3) Arc Length Association Method. All are available in our software, and we use all of them in our in-house asset allocations.

1. Estimation with Missing Data

An important consideration when dealing with financial data is how little of it is often available. Quantitative managers are aware that many complicated processes are happening in markets, and they want to capture and estimate as many of these as possible. Doing this requires using all available information, but often data streams start at different dates. Other times, data may not be available for certain time periods for other reasons. Since estimation for the purposes of optimization needs to render summary output for all the assets, usually in the form of expected returns, standard deviations, and correlations, and complete-data estimation procedures usually assume a block of data with equal numbers of observations, some technique for dealing with missing data is necessary in order to make full use of all available data. Through the years many different approaches have been proposed to handle missing data. Naïve approaches include filling in missing values with estimates, or using all available observations for each separate estimand. These methods produce biased estimates at best and sometimes lead to inconsistent results, unsuitable for optimization.

The current best-practice missing data algorithm for estimating mean and variance is a full-information maximum likelihood estimation (MLE) technique. It makes full use of all the information in the data and always produces coherent estimates. Basically, the estimates on variables with missing data are updated with information from other variables to reflect the missing time periods via rigorous statistics. In more detail, MLE uses the complete set of observed data to holistically estimate mean and variance, taking advantage of all linear relationships among the variables to consistently estimate all of the means, standard deviations, and correlations across the entire time period spanned by the dataset. In addition, estimates prepared by MLE are always positive-definite and do not cause any numerical issues in subsequent steps of the analysis. In cases with complicated patterns of missing data, the Expectation-Maximization (EM) Algorithm performs this calculation, but in simpler cases when missing data can be arranged in an order such that each successive variable's missing observations is a subset of the last, MLE can be performed non-iteratively based on a conditional factoring of the full likelihood formula. New Frontier's estimation software includes both and automatically picks the best choice for the available data. This method is the most suitable statistical practice for handling missing data to ensure that the final allocations make the best possible use of the available data.

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About New Frontier

New Frontier is a Boston-based institutional research and investment advisory firm specializing in the development and application of state-of-the-art investment technology. Founded in 1998 by the inventors of the world's first broad spectrum, patented, provably effective portfolio optimization process, the firm continues to pioneer new developments in asset allocation and portfolio selection. Based on practical investment theory, New Frontier's services help institutional investors, across the globe, to select and maintain more effective portfolios.

More information is available at **www.newfrontieradvisors.com**.

2. The t distribution

The Student t distribution is the sampling distribution of choice for obtaining plausible mean-variance assumptions within each resampled case in Michaud optimization. The t distribution properly uses the input information to take account of the unknown nature of the true mean and variance. It is one of the most famous distributions with heavy tails, and is better than the normal distribution at capturing extreme events. Thus it captures some extreme upside and downside scenarios that may be missed when using only the normal distribution. In practical terms, simulation studies have shown it to produce better performance out-of-sample than the normal distribution, especially in lower-risk portfolios on the frontier. Some of our simulation studies have shown as much as a 30 bp improvement in out-of-sample return at similar levels of risk, for less risky frontier portfolios. These performance gains are likely to be additive to other performance gains.

3. Subdividing Efficient Frontiers by Arc Length

When mean-variance frontier portfolios are averaged together to create a Michaud frontier, points must be selected to represent each frontier in the final average. The easiest method to implement by far divides the frontier by equally spaced expected returns. However, dividing each frontier into segments of equal arc length, while more costly computationally, produces better out-of-sample outcomes. Some simulated frontiers which curve in mean-variance space may exhibit better frontier coverage with the arc length method. The range of optimal portfolios covers the frontier more evenly, rather than clustering at the low-risk end of the frontier. Our simulation studies have shown modest, but statistically significant, gains of 5-10 bp of expected return for similar risk levels over most of the frontier using the arc length portfolio spacing over the mean portfolio spacing.

Conclusion

Research is one of the key pillars of New Frontier Advisors. We constantly strive to investigate new quantitative methods from statistics, finance, and other disciplines, evaluate and build on them, and recommend the best ways to enhance performance. The benefits of this research appear in our models as well as in our software. The three procedures briefly described here are but a few of many that we have examined and currently implement. Each method may yield only modest gains, but by combining methods, larger gains in expected performance can be realized. By choosing New Frontier, investors automatically gain access to expert vetting and implementation of the latest in quantitative financial research.

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